Notes on Frame Buckling

Kirk Martini
University of Virginia
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Introduction

The following notes include several examples of simple frame buckling problems which illustrate some of the assumptions and limitations of designing columns in frames using alignment charts. The examples emphasize the point that when a column is part of a frame with moment-resisting connections, the column no longer buckles as an isolated element. Buckling must be understood as a frame-wide phenomenon.

The examples are based on computer models using the *Arcade* program. The model files are included with this package and each example identifies the file name. It should be useful to run the model and see the behavior as you read the example.

Example 1: Braced frame with uniform beams and columns

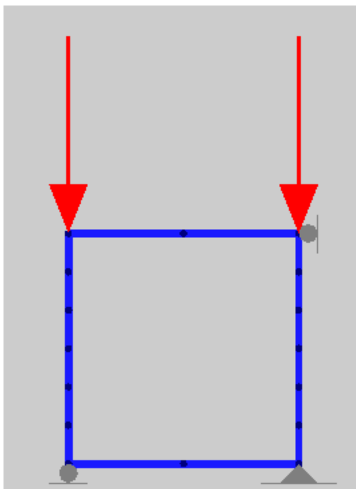

Figure 1.1. Square frame with uniform properties. *(col-buckle-frame/example-1)*

Figure 1.1 shows an Arcade model of a frame with the following properties.

column height = bay width = 10 ft.

E = 29,000 ksi

$I_{col} = I_{bm} = 10 \text{ in}^4$

Each column is modeled with six elements in a straight configuration, each beam is modeled with two elements.

Referring to figure C-C2.2 of the AISC LRFD code [AISC 2001, p. 16.1-191], the G ratio for top and bottom of each column is 1.0, since the columns and beams have equal moment of inertia and length. From the alignment chart, the effective length factor k = 0.77. The buckling load for each column is then

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 \left(29000^{ksi}\right)\left(10^{in^4}\right)}{\left((0.77)\left(120^{in}\right)\right)^2} = 335^{K}$$

Running the Arcade model of this example shows that the frame buckles at a load of 342 kips, which corresponds to an effective length factor of 0.762, quite close to the value of 0.77 taken from the alignment chart. Values from the chart are clearly approximate given the way that numbers are read from the logarithmic scale.

The following discussion and example Arcade models illustrate a few important points about frame buckling and the base assumptions of the alignment chart.

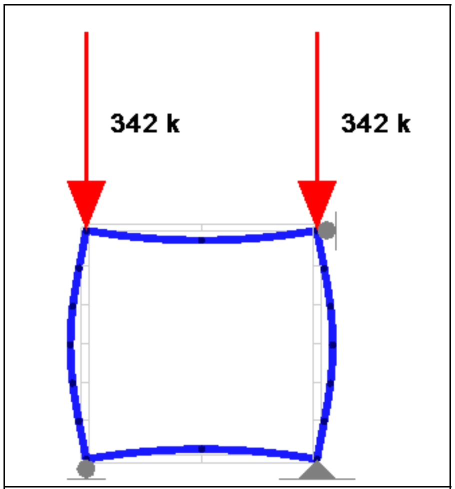

Figure 1.2. Buckled shape of square frame with uniform properties.
(col-buckle-frame/example-1)

Alignment charts assume beams in braced frames buckle in single curvature. One of the base assumptions of the alignment charts for braced frames is that end moments of the beams are equal in magnitude, producing single curvature bending [AISC 2001, p. 16.1-190]. This single curvature bending of girders is clear in the deflected model.

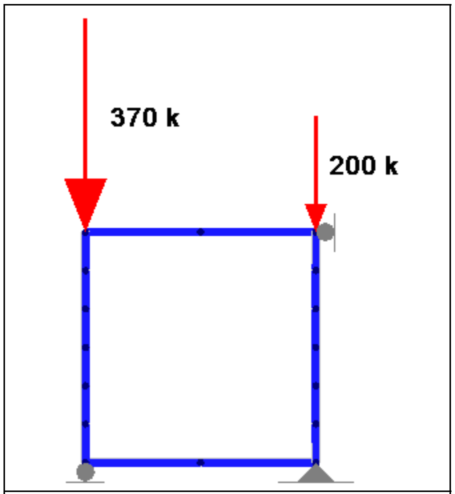

Figure 1.3. When one column is loaded below the critical load of 342 kips, the other column can be loaded above the critical load without buckling
(col-buckle-frame/example-1a)

Alignment charts assume columns buckle simultaneously. Figure 1.3 shows the same frame with a different load distribution. The right column is loaded with 200 kips, significantly less than the column's critical load of 342 kips, while the left column is load to 370 kips, 8 percent more than the critical load. Since the right column is loaded below its critical load, some of its stiffness is available to resist the overall buckling of the frame, which allows the left column to carry more than its critical buckling load.

In this respect, the alignment charts are conservative, because it is unlikely that all columns in a real frame would receive the Euler buckling load simultaneously.

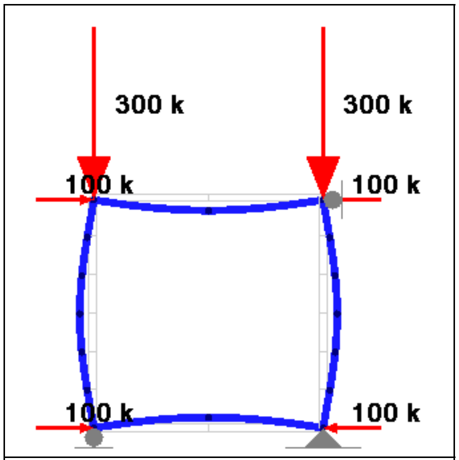

Figure 1.4. When the girders are in compression, the columns buckle at a lower load than the alignment chart predicts. *(col-buckle-frame/example-1b)*

Alignment charts assume that the beams are not in compression. Figure 1.4 shows the same frame with 100-kip compression forces added to each beam. With that compression force, the frame buckles when the vertical loads reach 300 kips, about 14% less than when the beams are not in compression.

In this respect, the alignment charts are not conservative, because it is possible that some beams would have compression, depending on the nature of the loading. In cases where there is significant compression in the girders, it would be inappropriate to use the alignment charts alone without accounting for the compression.

Example 2: Braced frame with stiffer columns

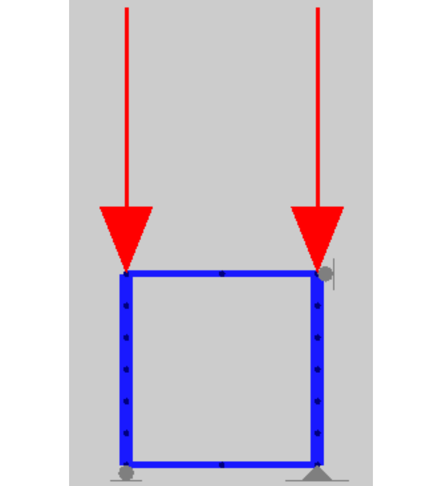

Figure 2.1. Square frame with colums stiffer than beams. *(col-buckle-frame/example-2)*

Figure 2.1 shows an Arcade model of a frame similar to that of example 1, except that the moment of inertia for the columns has been doubled (indicated graphically in the model by drawing the columns wider).

The properties of the frame are as follows:

column height = bay width = 10 ft.

E = 29,000 ksi

$I_{col} = 10\ in^4$

$I_{bm} = 20\ in^4$

The alignment chart G ratio for top and bottom of each column is 2.0, since the columns and beams have equal length, but the columns have twice as much moment of inertia. From the alignment chart, the effective length factor k = 0.855. The buckling load for each column is then

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 \left(29000^{ksi}\right)\left(20^{in^4}\right)}{\left((0.855)\left(120^{in}\right)\right)^2} = 544^K$$

Running the Arcade model of this example shows that the frame buckles at a load of 558 kips, which corresponds to an effective length factor of 0.844, again reasonably close considering the approximation involved in reading the scale. The key point of this example is the following:

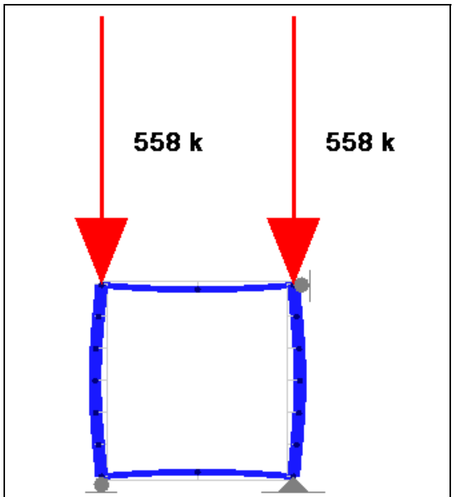

Figure 2.2. Buckled frame. *(col-buckle-frame/example-2)*

Doubling the stiffness of the columns alone does not double the buckling load. Doubling the moment of inertia of the columns increased the buckling load of the Arcade model from 342 kips to 558 kips, an increase of 63%. The increase in buckling load was not proportional to the increase in moment of inertia because increasing the moment of inertia changed the buckled shape. The stiffer columns impose greater rotations on the beams, moving the column inflection points farther apart and increasing the column's effective length. This increase in effective length reduces buckling resistance and counteracts the additional resistance provided by the increased moment of inertia. To double the buckling load, it is necessary to double the moment of inertia of both the columns *and* the beams.

Example 3: Unbraced frame

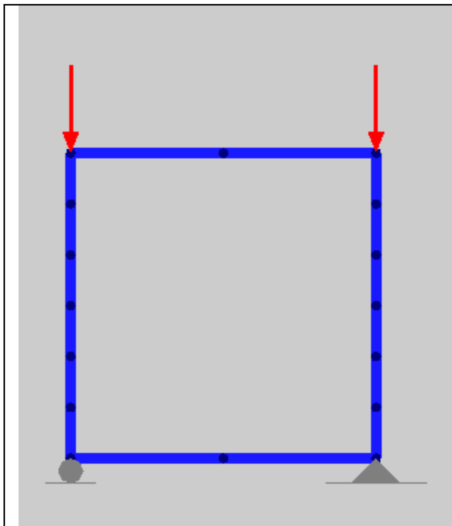

Figure 3.1. Square frame not braced against sidesway. *(col-buckle-frame/example-3)*

Figure 3.1 shows an Arcade model of a frame similar to that of example 1, except that the frame is no longer braced against sidesway.

The properties of the frame are as follows:

column height = bay width = 10 ft.

E = 29,000 ksi

$I_{col} = I_{bm} = 10\ in^4$

The alignment chart G ratio for top and bottom of each column is 1.0. From the alignment chart for sidesway uninhibited, the effective length factor k = 1.32. The buckling load for each column is then

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 (29000^{ksi})(10^{in^4})}{((1.32)(120^{in}))^2} = 114^K$$

Running the Arcade model of this example shows that the frame buckles at a load of 116 kips, which corresponds to an effective length factor of 1.31 The key point of this example is the following:

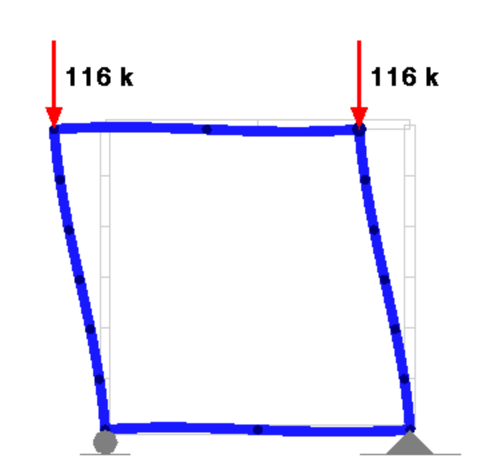

Figure 2.2. Buckled frame. *(col-buckle-frame/example-3)*

Allowing sidesway significantly reduces the buckling load. For the braced case the column buckling load was 342 kips, compared with 116 kips for the unbraced case. The buckling load for the unbraced frame is 34% of the buckling load for the braced frame: an extremely significant reduction.

Alignment charts assume beams of unbraced frames buckle in double curvature. One of the base assumptions of the alignment charts for unbraced frames is that the end moments of the beams are equal in magnitude, producing double curvature bending [AISC 2001, p. 16.1-190]. This double curvature bending of girders is clear in the deflected model.

Example 4: Unbraced frame with stiffer columns

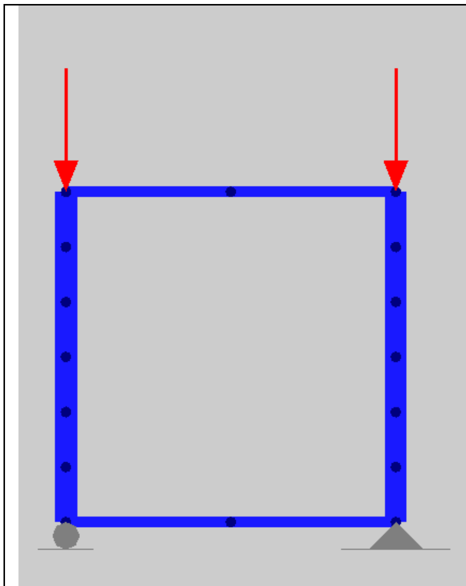

Figure 4.1. Square frame with columns twice as stiff as beams. *(col-buckle-frame/example-4)*

Figure 4.1 shows an Arcade model of a frame similar to that of example 3, except that the moment of inertia for the columns has been doubled (indicated graphically in the model by drawing the columns wider).

The properties of the frame are as follows:

column height = bay width = 10 ft.

E = 29,000 ksi

$I_{col} = 20\ in^4$

$I_{bm} = 10\ in^4$

The alignment chart G ratio for top and bottom of each column is 2.0, since the columns and beams have equal length, but the columns have twice as much moment of inertia. From the alignment chart, the effective length factor k = 1.58. The buckling load for each column is then

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 \left(29000^{ksi}\right)\left(20^{in^4}\right)}{\left((1.58)\left(120^{in}\right)\right)^2} = 159^K$$

Running the Arcade model of this example shows that the frame buckles at a load of 159 kips. The key point of this example is the following:

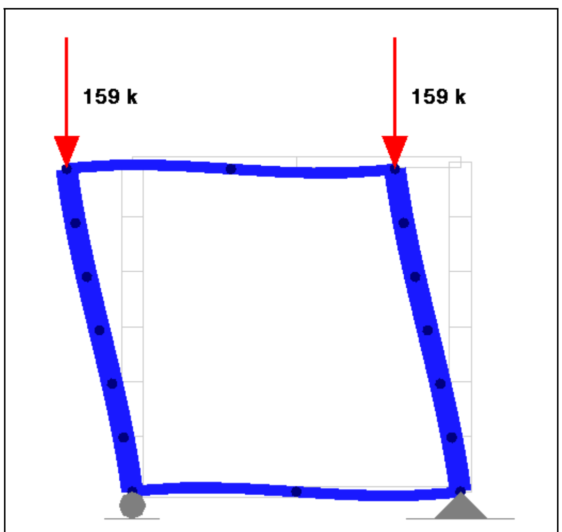

Figure 4.2. Buckled frame. *(col-buckle-frame/example-4)*

In an unbraced frame, increasing the moment of inertia of columns alone has less effect than on a braced frame. In this unbraced frame example, doubling the moment of inertia of the columns increased the buckling load from 116 kips to 159 kips, an increase of 37 percent. Recall in the braced frame that when the column stiffness was doubled, the buckling load of the frame increase 63 percent. Increasing the column stiffness is clearly less effective in the sway frame than in the non-sway frame.

The difference between the braced and unbraced case concerns the effect of increasing column stiffness on effective length. In both cases, increasing the column stiffness leads to an increase in effective length. This increase in effective length decreases buckling resistance, which offsets the increase in buckling resistance provided by the stiffer column; this is why doubling the column stiffness does not double the buckling load. But, in the braced case there is a limit to the increase in effective length that occurs when the column is stiffened. As the column moment of inertia increases, the effective length approaches 1.0. For the case of an extremely stiff column in a braced frame, doubling the column stiffness nearly will double the buckling load.

In contrast, in the unbraced case, there is no limit to the effective length. As column stiffness is increased, the effective length continues to increase without limit so that increases in column stiffness produce diminishing returns in the increase in buckling resistance.

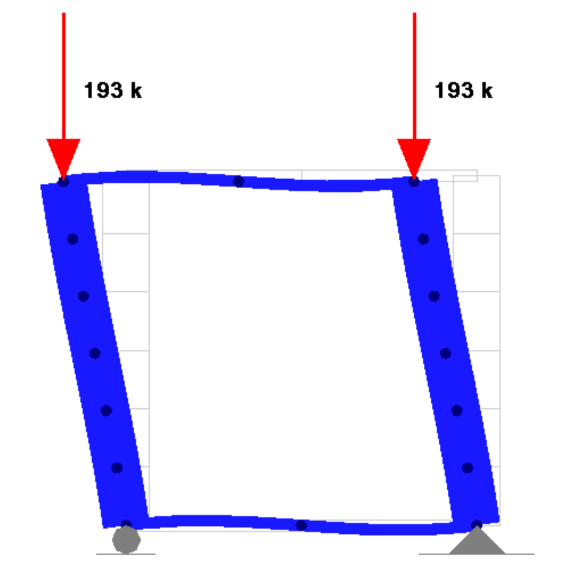

Figure 4.3. Buckled shape where columns are four times stiffer than girders. *(col-buckle-frame/example-4a)*

Figure 4.3 shows the buckled shape of the frame from figure 4.1 where the column stiffness has been quadrupled (I_{col} = 10 in^4, I_{bm} = 40 in^4). The alignment chart G ratio for top and bottom of each column is 4.0. From the alignment chart, the effective length factor k = 2.02. The buckling load for each column is then

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 \left(29000^{ksi}\right)\left(40^{in^4}\right)}{\left((2.02)\left(120^{in}\right)\right)^2} = 195^K$$

The Arcade model buckles at approximately 193 kips. The deflected shape shown in the figure makes clear that the buckling results primarily from the deformation of the beams rather than the columns, which rotate as nearly rigid bodies.

Summary

Assumptions of alignment charts:

- **Alignment charts assume columns buckle simultaneously**. If some columns in a frame a loaded below their Euler buckling load, then other columns in the frame may be able to carry more than their Euler buckling load. In this respect, the alignment charts are conservative, because it is unlikely that all columns in a real frame would receive the Euler buckling load simultaneously.
- **Alignment charts assume that the beams are not in compression**. When beams are in compression, it reduces the effectiveness of the beams in restraining column end rotation, and the columns buckle at a lower load. In this respect, the alignment charts are not conservative, because it is possible that some beams could have compression, depending on the nature of the loading. In cases where there is significant compression in the beams, it would be inappropriate to use the alignment charts alone.

General frame behavior

- **In a frame with moment-resisting connections, buckling is a phenomenon of the complete frame-column assembly**. When a column in a moment-connected frame buckles, other members connected to the column also buckle, so the buckling of the column cannot be considered in isolation. This is why doubling the stiffness of the columns alone does not double the column buckling load.
- **Allowing sidesway significantly reduces the buckling load.** In the example problems, comparing braced and unbraced versions of frames with equal dimensions and members found that the buckling load of the unbraced version was 34% that of the braced version. Buckling is a much greater consideration in unbraced frames that in braced.
- **In an unbraced frame, increasing the moment of inertia of columns alone the does little to increase buckling resistance.** In an unbraced frame, as the stiffness of the columns is increased, the effective length of the column also increases without limit, so there are diminishing gains in buckling resistance with increases in column stiffness. In the design of an unbraced frame, the stiffness of beams and frames should be considered together. Stiffening beams may often be the most effective way to increase the buckling resistance of an unbraced frame.